Section 2.4 The Product & Quotient Rule

Find $f'(x)$ of

$$f(x) = (x+3x^2)(2x^3+9)$$

Product Rule

$$\frac{d}{dx} = f'(x)g(x) + f(x)g'(x)$$

$$= \overbrace{(x+3x^2)}^{f(x)}\overbrace{(2x^3+9)}^{g(x)}$$

$$= (1+6x)(2x^3+9) + (x+3x^2)(6x^2)$$

$$= 2x^3+9+12x^4+54x+6x^3+18x^4$$

$$\boxed{\frac{d}{dx} = 30x^4+8x^3+54x+9}$$

$$f(x) = \left(\frac{1}{x} + 1\right)(x-1)$$

$$f(x) = (x^{-1}+1)(x-1)$$

$$f(x) = (1 - x^{-1} + x - 1)$$

$$f'(x) = x^{-2} + 1$$

$$\boxed{f'(x) = \frac{1}{x^2} + 1}$$

$$f(x) = (x^{-1}+1)(x-1)$$

$$= (-x^{-2})(x^{1} - 1) + (x^{-1}+1)(1)$$

$$= \cancel{-x^{-1}} + x^{-2} \cancel{+x^{-1}} + 1$$

$$= x^{-2} + 1$$

$$\boxed{= \frac{1}{x^2} + 1}$$

$$= (x^2+1)(x^3+7)(4x-2)$$

$$\frac{d}{dx} = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

Quotient Rule

$$\frac{x-1}{2x+3} \qquad \frac{f(x)}{g(x)}$$

$$\frac{d}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx} = \frac{(2x+3)(1) - (x-1)(2)}{(2x+3)^2}$$

$$\frac{d}{dx} = \frac{2x+3-2x+2}{(2x+3)^2}$$

$$\boxed{\frac{d}{dx} = \frac{5}{(2x+3)^2}}$$

$$f(x) = \frac{2x^2-4x+3}{2-3x}$$

$$= \frac{(2-3x)(4x-4)-(2x^2-4x+3)\overset{(-3)}{\cancel{(2-3x)}}}{(2-3x)^2}$$

$$= \frac{(8x-8-12x^2+12x+6x^2-12x+9)}{(2-3x)^2}$$

$$= \frac{-6x^2+8x+1}{(2-3x)^2}$$

$$f(x) = \frac{3-1/x}{x+5}\,\frac{(x)}{(x)}$$

$$f(x) = \frac{3x-1}{x^2+5x}$$

$$f'(x) = \frac{(x^2+5x)(3)-(2x+5)(3x-1)}{(x^2+5x)^2}$$

$$f'(x) = \frac{3x^2+15x-6x^2+2x-15x+5}{(x^2+5x)^2}$$

$$f'(x) = \frac{-3x^2+2x+5}{(x^2+5x)^2}$$

$$y = \frac{x^2+3x}{6} \longrightarrow y = \frac{1}{6}(x^2+3x) \longrightarrow y' = \frac{1}{6}(2x+3) = \frac{2x+3}{6}$$

$$y = \frac{5x^4}{8} \longrightarrow y = \frac{1}{8}(5x^4) \longrightarrow y' = \frac{1}{8}(20x^3) \longrightarrow \frac{5x^3}{2}$$

$$y = \frac{-3(3x-2x^3)}{7x} \longrightarrow \frac{-9x+6x^3}{7x} \longrightarrow \frac{-9+6x^2}{7} \longrightarrow \frac{1}{7}(-9+6x^2)$$

$$y = \frac{9}{5x^2} \longrightarrow \frac{1}{5}(9x^{-2})$$